Puzzler: S'mores n' more

1. How much energy is needed?

Q. Prove to yourself that the chocolate needs about 130 J to melt using Eq. 2. How impatient could you be and still have it work? Note this assumes what about the temperature of the marshmallow?

First, let's prove that we need ~2.1 kJ to melt the marshmallow.

Starting with equation 1:

$$Q = c \cdot m \cdot \Delta T$$

For the marshmallow we have:

$$Q_{Marshmallow} = \left(2 \frac{\text{kJ}}{\text{Kg °C}}\right) \cdot \left(0.35 \frac{\text{g}}{\text{cm}^3} \cdot \pi \left(\frac{0.025m}{2}\right)^2 \cdot 0.038\, m\right) \cdot (186\text{°C} - 23\text{°C}) = 2.1\, kJ$$

Now, the energy transfer using Eq 2:

$$Q = k \cdot A \cdot \Delta T \cdot t/L$$

(note that this assumes a constant temperature differential… meaning the marshmallow stays at 186 °C during the full minute, which is not really true….

Plugging in values:

$$Q_{Marshmallow} = \left(0.5 \frac{\text{W}}{m\ \text{°C}}\right) \cdot \left(\left(\frac{0.136\ \text{m}}{2}\right) \cdot 0.054m\right) \cdot (30\text{°C} - 23\text{°C}) \cdot \frac{60\ s}{0.006\ m} = 130\, J$$

(Note: I used the area of the whole piece of chocolate. This assumes the marshmallow was smooshed into the chocolate)

This is the amount of energy transferred through the chocolate. This isn't exactly what we want but it does tell us, assuming no phase transitions, how fast the energy moves from the marshmallow through the chocolate. If we ignore the time it takes the energy to transfer we can think about how much energy is needed to raise the temperature of the chocolate to it's melting point. So according to eq 1, how much energy is needed to raise the:

$$Q_{chocolate} = \left(1.8 \frac{\text{kJ}}{\text{Kg °C}}\right) \cdot \left(1300 \frac{\text{kg}}{\text{m}^3} \cdot \left(\frac{0.136\ \text{m}}{2}\right) \cdot 0.054m \cdot 0.006\, m\right) \cdot (30\text{°C} - 23\text{°C}) = \ 360\, kJ$$

2. What would the final temperature be?

Challenge: Now, ignore heat transfer and focus just on the total energy gains by the campfire. Assuming the graham crackers are infinitely good insulators, have an extremely small specific heat, and we wait an infinitely long time, what will the final temperature ultimately be?

The key to answering this part is to realize we have a closed system. If the graham crackers are perfect insulators, have an ignorable heat capacity (~ 0) and is allowed an infinitely long period of time to equilibrate, the heat gained by the chocolate is lost by the marshmallow to an unknown temperature t.

$$Q_{chocolate\ gain} = Q_{marshmallow,loss}$$

$$Q_{chocolate\ gain} = \left(1.8\,\frac{\mathrm{kJ}}{\mathrm{Kg\ °C}}\right)\cdot\left(1300\,\frac{\mathrm{kg}}{\mathrm{m^3}}\cdot\left(\frac{0.136\ \mathrm{m}}{2}\right)\cdot 0.054m\cdot 0.006\ m\right)\cdot(t-23\mathrm{°C}) =$$

$$Q_{marshmallow,loss} = \left(2\,\frac{\mathrm{kJ}}{\mathrm{Kg\ °C}}\right)\cdot\left(0.35\,\frac{\mathrm{g}}{\mathrm{cm^3}}\cdot\pi\left(\frac{0.025m}{2}\right)^2\cdot 0.038\ m\right)\cdot(186\mathrm{°C}-t)$$

If you solve for the equilibrium temperature t you arrive at ~56 °C. Note that this ignores the energy required to actually melt the marshmallow and chocolate but for the sake of this back of the envelop question, there is plenty of energy in the marshmallow to raise the temperature of the chocolate to it's melting point.