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Physics Puzzler: The Physics of Pumpkin Chucking Part 1

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For an adiabatic expansion

$$PV^{\gamma} = P_0^{\gamma} V_0^{\gamma}$$

wherein, $\gamma = 7/5$, the pressures are absolute, V_0 is the volume of the reservoir

$$V = V_0 + AL(t)\ ;\ 0 \le L(t) \le L_m$$

L_m is the length of the launch tube, and P_0 is the initial pressure in the reservoir. Thus,

$$P = P_0\left(1 + \frac{AL}{V_0}\right)^{-\gamma}$$

The work done by the expanding gas for a differential step is:

$$dW_p = \left[P_0\left(1 + \frac{AL}{V_0}\right)^{-\gamma} - p_a\right]AdL$$

so after traveling a distance L_1:

$$W_p = P_0 V_0 \int_0^{u_1} (1+u)^{-\gamma} du - p_a A L_1$$

wherein, $u = AL/V_0$. Thus

$$W_p = \frac{5}{2} P_0 V_0 \left(1 - \frac{1}{(u_1+1)^{2/5}}\right) - p_a A L_1$$

or

$$W_p = \frac{5}{2} P_0 V_0 \left[\left(1 - \frac{1}{\left(\frac{AL_1}{V_0} + 1\right)^{2/5}}\right) - \frac{2}{5}\frac{p_a A L_1}{P_0 V_0}\right]$$

With $V_1 = V_0 + AL_1$, $\tilde{p}_0 = p_0 p_a$ and $\tilde{V}_1 = V_1/V_0$

$$W_p = \frac{5}{2} p_a V_0 (\tilde{p}_0 + 1)\left[\left(1 - \frac{1}{\tilde{V}_1^{\,2/5}}\right) - \frac{2}{5}\frac{\tilde{V}_1 - 1}{\tilde{p}_0 + 1}\right]$$

Assuming no frictional drag in the launch tube,

$$W_p = \frac{1}{2}\rho\frac{4}{3}\pi R^3 v_1^2$$

$$\rightarrow v_1^2 = \frac{15}{4\pi}\frac{p_a V_0}{\rho R^3}(1 + \tilde{p}_0)\left[\left(1 - \frac{1}{\tilde{V}_1^{\,2/5}}\right) - \frac{2}{5}\frac{\tilde{V}_1 - 1}{1 + \tilde{p}_0}\right]$$

For

6” and 4” tubes, $\tilde{V}_1 = 13/9$; $p_0 = 80\text{psi}$, $\tilde{p}_0 = 11/2$

$R = 1/20\text{m}\ ;\ L_0 = 81" \approx 2\text{m}\ ;\ V_0 \cong \pi/100\text{m}^3\ ;\ \rho \cong 10^3\ \text{kg/m}^3$

$$v_1^2 \cong \frac{17}{20}\frac{p_a V_0}{\rho R^3}$$

$$v_1 \approx \sqrt{\frac{17}{20} 20^3 \frac{(10^5\ \text{N/m}^2)(\pi\ /\ 100\ \text{m}^3)}{(10^3\ \text{kg/m}^3)(\text{m}^3)}} \sim 150\frac{\text{m}}{\text{s}}$$

The measured speed was about 80 m/s. Assuming that this difference is due to a frictional force. Let us try to estimate the frictional coefficient.

Assuming the spherical projectile deforms to a cylinder, $4\pi R^3\ /\ 3 = \pi R^2 \rightarrow l_0\ /\ R = 4\ /\ 3$. The pressures on opposing faces are $p + p_a$ and p_a. The pressure in this cylinder will decrease linearly with distance along the
cylinder:

$$p_{in} = (p + p_a) - \frac{p}{l_0} l$$

The gauge pressure in the fluid will cause differential normal force

$$dN = p\left(1 - \frac{l}{l_0}\right) 2\pi R dl$$

and a differential drag force

$$df_k = \mu_k p\left(1 - \frac{l}{l_0}\right) 2\pi R dl$$

Integrating

$$f_k = 2\pi\mu_k R p \frac{1}{2} l_0 = \pi\mu_k p R l_0$$

The work done by this force is

$$W_f = -\pi\mu_k R l_0 \int p dL = -\pi\mu_k R l_0 \frac{1}{A} W_p = -\mu_k \frac{l_0}{R} W_p$$

With $\frac{l_0}{R} = 4/3$, the net work done is

$$W_{net} = W_p + W_f = W_p\left(1 - \mu_k \frac{4}{3}\right) = \frac{1}{2}\rho\frac{4}{3}\pi R^3 v_1^2$$

Thus

$$\mu_k = \frac{3}{4}\left(1 - \frac{2}{3}\frac{\rho\pi R^3 v_1^2}{W_p}\right)$$

$$= \frac{3}{4}\left(1 - \frac{4}{15}\frac{\rho\pi R^3 v_1^2}{p_a V_0(\tilde{p}_0 + 1)}\frac{1}{\left(1 - \frac{1}{\tilde{V}_1^{\,2/5}}\right) - \frac{2}{5}\frac{\tilde{V}_1 - 1}{\tilde{p}_0 + 1}}\right)$$

With $v_1 = 80$ m/s, $\mu_{k,min} \sim 0.6$

Additional comments and partial solution:

Use your calculated launch speed for a smooth grapefruit, the density of air (1.2 kg/m^3), and the dynamic viscosity of air (18 microPascal·seconds). What is your estimate of the air drag coefficient in this case?

B&M: with a launch speed of 80 m/s and a projectile diameter of 4", the Reynolds number is … about 5x10^5: i.e. very close to the dip in the reference.

Pumpkins have a larger diameter and they have surface roughness, so they can have a low air drag coefficient at lower speeds.

Since grapefruits are much squishy than pumpkins, they deform in response to the pressure difference across them while in the launch tube. This causes them to have a tight seal with the walls of the launch tube. Compared to a pumpkin, this reduces the loss of pressurized air around the projectile, blow-by, but increase the friction between the surface of the projectile and the walls of the launch tube. However, the natural oil in the skin of the grapefruit helps mitigate the latter. For a pumpkin, the blow-by reduces the launch speed, but it also has another effect. Since any real pumpkin is not entirely symmetrical, the blow-by can cause the pumpkin to start spinning. This can cause a "curve ball" effect, that is also important for spinning golf balls, and can cause the pumpkin to curve upward, or in a variety of other directions. This occasionally happens with grapefruits.

B&M It is very unlikely that the expansion is isothermal. After each launch of a grapefruit, there is a small puff or cloud of "air" exiting the launch tube. These are, of course, tiny ice crystals. However, reducing the temperature from about 300 K to about 270 K is only a 10% change in temperature. The snug fitting, but very rigid, Wabash projectiles should have little friction. This is consistent with them not seeing any frictional effect! Depending on the kind of valve employed, the flow coefficient effect employed at Wabash could be very important, as it was for their cannon. Valves with small internal orifices will provide low flows, but the entire geometry of a valve contributes to its flow coefficient. By using a large deformable diaphragm, I had hoped to have good air flow. Some with similar spud cannons have use a movable plug.