# Preliminary Examination of Uncertainties due to Parton Distribution Functions in Far-Forward Neutrino Production at the Large Hadron Collider

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**Abstract.** A large flux of neutrinos is expected in the forward direction of the pp collisions [1, 3] at the Large Hadron Collider (LHC) at CERN. Several experiments have recently been proposed at CERN to detect these neutrinos and discussion has started on the possibility of building a Forward Physics Facility grouping many of them. Among the others, the FASER-nu [2] proposal consists of a lead+emulsion neutrino detector at a distance of 480 m from the ATLAS interaction region along the tangent to the LHC beamline. Old calculations of neutrinos rates in the forward direction were done to leading order in the QCD perturbative series. We have included next-to-leading order (NLO) terms in our calculation. We have also studied the effect of incorporating a non-perturbative Gaussian intrinsic  $k_T$ . This  $k_T$  effect mimics the contribution from missing higher-order terms in QCD calculations. We present the study of uncertainties due to scale variations and Parton Distribution Function (PDF) variations in the production rate of  $D_s^{\pm}$  and tau neutrinos in the far-forward Production at the Large Hadron Collider. We compare our predictions with the LHCb data with  $D_s^{\pm}$  production in the rapidity range of [2-4.5] at the LHC. In our studies, for which we consider various modern PDF sets, we also include the comparison of predictions obtained by different dynamical central scale assumptions.

# INTRODUCTION

Neutrinos are one of the fundamental particles that make up the universe. They are one of the most abundant particle in the universe and are also the least understood. The neutrino is a subatomic particle that resembles an electron but is electrically neutral and has a very small mass compared to other elementary particles. Neutrinos belong to the lepton family in the standard model. Lepton is a family of particles that doesn't experience strong interactions. Out of the four fundamental forces, neutrinos interact only through the weak force and gravity. As the weak nuclear force is very short-range and gravitational interaction of the neutrino is extremely weak, they rarely interact with normal matter. There are trillions of neutrinos passing through us every second.

Neutrinos come in three flavors: electron, muon, and tau n eutrinos. They are labeled after their charged partners within the Standard Model. In charged current neutrino interactions in matter, neutrinos of a given type result in the emission of their charged partner. Out of the three flavors, tau neutrinos are the least studied as there is not sufficient data available to study them. The collision of proton beams at 14 TeV at the Large Hadron Collider (LHC) produces a large flux of hadrons. The hadrons produced in the proton/proton collisions decay further into neutrinos. In particular, the decays of  $D_s^{\pm}$  and B mesons produce a large flux of tau neutrinos. A new set of experiments have been proposed at CERN [2]. This paper is about the calculation of the flux of neutrinos in the forward direction at the LHC, along with the flux we study the uncertainties in our predictions caused by seven scale variations and Parton Distributions Functions.

The forward region can be understood with the kinematic variable called pseudo-rapidity. Pseudo-rapidity ( $\eta$ ) is a geometric quantity and is a function of the angle  $\theta$  with respect to the collision axis as seen in Fig.1. In this project, we are exploring the region with  $\eta > 6.87$  which approximately covers the forward detector coverage for the proposed experiments at CERN.

$$\eta = -\ln(\tan(\frac{\theta}{2})) \tag{1}$$

There are two problems that arise in the Quantum Chromodynamics (QCD) perturbation theory that is used to calculate meson production in pp collisions: ultra-violet (UV) and infrared (IR) divergences. To includes the higher-order term in perturbation theory, we come across Feynman graphs with closed loops, that are associated with unbounded energy. Because of unconstrained energy, the integral associated with such Feynman diagrams tend to diverge. Such divergences are called UV divergences. These divergences are not physical. The UV divergence are cured by introducing the renormalization factor ( $\mu_R$ ). We also encounter Feynman diagrams which include massless particles of energy



**FIGURE 1:** (a) The relation between  $\eta$  and  $\theta$ : An angle of zero is along the beam axis. Generally, particles in the high pseudorapidity regime escape through space in the detector along with the beam axis (forward direction), and this project includes the study of tau neutrino in the forward direction. (b) Proton-proton collision: Gluons from colliding proton interact during the collision. Gluon-Gluon interaction produce a quark and anti-quark which further produces hadrons like  $D_s^{\pm}$  meson.  $D_s^{\pm}$  meson decay into tau neutrinos.

approaching zero. The integral of Feynman diagrams including massless particles with zero energy also diverge. The divergence due to massless particles is known as Infrared(IR) divergences. The problem of IR divergence is solved by introducing the factorization factor ( $\mu_F$ ). The renormalization and factorization scales ( $\mu_R, \mu_F$ ) are defined to be the factors ( $N_R, N_F$ ) multiplied by the transverse mass  $m_T$ . We will use use two different definitions for the transverse mass:

$$m_T = \sqrt{m_c^2 + p_T^2}$$
 and  $m_T = \sqrt{4m_c^2 + p_T^2},$ 

where  $p_T$  is the magnitude of the transverse momentum of the charm quark and  $m_c$  is the charm quark mass.

The other variable in our calculation is the Parton Distribution Function (PDF). Parton, name given by Richard Feynman, refers to particle (quarks and gluons) constituents within the protons, neutrons and other hadrons. In the proton/proton collision, what actually happen is that these partons collide with each other as seen in Fig.1a. The colliding parton carries a fraction of the momentum of the proton. Parton distributions functions are momentum distribution functions of the partons within the proton. They are probability densities to find a parton with momentum with fraction x at an energy scale of  $\mu^2$ . We can calculate the hadronic cross section using these PDF.

# THEORY

The most important concept in collisions of subatomic particles is their cross section. The word cross section is first introduced in mathematics as a intersection between a plane and a three dimensional object. In physics, the word cross section also has units of area, but comes from a different consideration. The cross section with units of area governs the probability that two particles will collide or interact to produce a certain outcome and is denoted by  $\sigma$ . For example, the total cross section for production of tau neutrinos could be written as:

$$\sigma = \frac{\text{\# of tau neutrinos produced per unit time}}{\text{Luminosity of protons per unit area per unit time}}$$
(2)

The other important term is Luminosity. In the above formula, it can be seen that Luminosity is the ratio of the number of events detected (N) in a certain time (t) to the cross-section. Using cross section and luminosity, we can

calculate the number of events (dN/dt events/sec) by the following:

$$dN/dt = L \times \sigma . \tag{3}$$

As mentioned earlier, we include NLO QCD corrections to the heavy-quark (HQ) production cross section. The HQ production cross section under perturbative QCD is as follow [4]:

$$E\frac{d^{3}\sigma}{dp^{3}} = \sum_{i,j} \int dx_{1} dx_{2} f_{i}^{H_{1}}\left(x_{1}, \mu_{F}^{2}\right) f_{j}^{H_{2}}\left(x_{2}, \mu_{F}^{2}\right) \left[ E\frac{d^{3}\hat{\sigma}_{ij}\left(x_{1}P_{H_{1}}, x_{2}P_{H_{2}}, p, m^{2}, \mu_{F}^{2}, \mu_{R}^{2}\right)}{dp^{3}} \right]$$

(4) where  $f_i^{H_1}(x_1, \mu_F^2)$  and  $f_j^{H_2}(x_2, \mu_F^2)$  are parton distribution functions (PDFs),  $\mu_F^2$  and  $\mu_R^2$  are factorization and renormalization scales, respectively. As we discussed before, we need to account for mean transverse momentum in our calculation. We use a Gaussian approximation for transverse momentum in 2 dimensions.

$$f(\vec{k_T}) = \frac{1}{\pi \langle k_T^2 \rangle} e^{-\frac{k_T^2}{\langle k_T^2 \rangle}}$$
(5)

After including the  $k_T$  effect and integrating over it, the heavy quark production cross section becomes:

$$E\frac{d^2\sigma}{dp_z d^2 p_T} = \int d^2 k_T \int d^2 p_T' f(\vec{k_T}) E\frac{d^2\sigma}{dp_z d^2 p_T'} \delta^2(\vec{p_T} - \vec{p_T'} - \vec{k_T})$$
(6)

The theoretical evaluation of the production of heavy quarks like charm has been studied for quite a long time, and is already implemented in a computer program called HVQ [5, 6] using the FORTRAN language. We used this program with some modifications to run simulations in our study. The HVQ code uses the Vegas algorithm [7] to calculate the integrals for the heavy quark production cross section. As there is no data available in the forward direction for production of particles, we use LHCb data for the production of  $D_s^{\pm}$  to compare with our theoretical predictions. We provide predictions by varying three parameters:  $k_T$ ,  $\mu_F^2$  and  $\mu_R^2$ . By varying these parameters, we tried to fit the data with our predictions using different transverse masses as discussed above. From the previous studies, we found that  $k_T = 0.7$  GeV fit well with the data [8, 9].



**FIGURE 2**:Comparison between our predictions and LHCb experimental data on double-differential cross section for  $D_s^{\dagger}$  production. Data and predictions for different y bins are shifted by 10<sup>-m</sup> where values of m = 0, 2, 4, 6 and 8. Fig. (a) refers to the central scale N<sub>R</sub> = 1.0, N<sub>F</sub> = 2.0 with with  $m_T = \sqrt{m_C^2 + p_T^2}$  with  $\langle k_T \rangle = 1.2$ . Fig. (b) shows to the central scale N<sub>R</sub> = 1.0, N<sub>F</sub> = 1.0 with  $m_T = \sqrt{4m_C^2 + p_T^2}$  with  $\langle k_T \rangle = 1.2$ . The colored portion shows the uncertainty band of seven scale variations.

The scale variations  $(N_R, N_F)$  in  $(\mu_R, \mu_F) = (N_R, N_F)m_T$ , where  $m_T = \sqrt{m_c^2 + p_T^2}$  are [(1,2), (0.5, 1), (2,4), (0.5,2), (1,1), (2,2), (1,4)] with (1,2) as central scale choice. The scales variations  $(N_R, N_F)$  in  $(\mu_R, \mu_F) = (N_R, N_F)m_T$ , where  $m_T = \sqrt{4m_c^2 + p_T^2}$  are [(1,1), (0.5,0.5), (2,2), (0.5,1), (1,0.5), (2,1), (1,2)] with (1,1) as central scale choice. By examining the scale variations, we find the best central scale assumptions of transverse mass with  $k_T$ . In our studies, we are using PROSA [10] PDF. The PDF are created using data from Deep Inelastic Scattering (DIS) of leptons and fit with different models and parameters. PROSA contains 40 variations that account for PDF fit uncertainty, PDF parameter uncertainty, PDF model uncertainty. The fit uncertainties come from the data that is used to create the PDF, parameter uncertainties arise from the parameter used to fit the data, and model uncertainties originate from the assumed model for the fit for the PDF. The total uncertainties are obtained by adding fit, model, and parameter uncertainties in quadrature.



#### RESULTS

**FIGURE 3.** These figures show the uncertainties in the calculation of cross section of tau neutrinos in the rapidity range from  $6.7 < \eta < 7.5$ ,  $7.2 < \eta < 8.7$ , and  $8.0 < \eta < 9.2$ . The selected rapidity ranges represent the forward region of the pp collision.

In Fig. 2, we compare our prediction of  $D_s^{\pm}$  double differential cross section with the LHCb data. By comparing

figures 2a and 2b, we conclude that transverse momentum  $m_T = \sqrt{m_c^2 + p_T^2}$  with  $k_T = 1.2$  GeV produces better prediction. We select the parameter,  $\langle k_T \rangle = 1.2$  GeV, and  $m_T = \sqrt{m_c^2 + p_T^2}$  with central scale of  $(N_R, N_F) = (1.0, 2.0)$ , to further study the uncertainties in PDFs for tau neutrino production in the forward region. Fig. 3 shows the uncertainties as bands around the central PDF assumption due to variations in PDFs in the cross section of tau neutrinos in the forward region of the pp collision. The majority of the uncertainties are due to model assumptions made in creating the PDF. These model assumptions vary from different collaborations who have produced these PDFs. The root of the problem arises from the lack of knowledge of the structure of the nucleon. To get better estimates of the production rate in the forward direction, we need better understanding of model assumptions for PDFs.

## CONCLUSION

Neutrinos are mysterious and everywhere around us. Understanding neutrinos is a challenging task as they rarely interact with matter. FASER-nu [2] experiment has been proposed to measure the flux of neutrinos in the forward direction at the LHC. This paper focused on flux uncertainties using seven scale variations and PDF uncertainties in the production of  $D_s^{\pm}$  and tau neutrinos from decays of these mesons. According to our results, the calculation of  $D_s^{\pm}$  double differential cross section with transverse momentum  $m_T = \sqrt{m_c^2 + p_T^2}$  is the best fit for the LHCb data. The tau neutrino flux uncertainties with  $m_T = \sqrt{m_c^2 + p_T^2}$  are comparably less than uncertainties with  $m_T = \sqrt{4m_c^2 + p_T^2}$ . The analysis of PDF uncertainties is still under way. According to our initial studies, the dominant part of the uncertainty is due to the PDF model itself. The studies will continue to examine the various elements of uncertainties due to the PDFs and find ways to reduce those uncertainties in our calculation.

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## REFERENCES

- 1. A. De Rujula and R. Ruckl, "Neutrino and muon physics in the collider mode of future accelerators" in *Proceedings of the ECFA-CERN* Workshop : Large Hadron Collider in the LEP tunnel (Lausanne, 1984) pp.571-596.
- 2. FASER collaboration, A. Ariga et al., Technical Proposal for FASER: ForwArd Search ExpeRiment at the LHC, 1812.09139.
- 3. K. Winter, "Detection of the tau-neutrino at the LHC" in ECFA Large Hadron Collider Workshop Proceedings.2., (Aachen, 1990) pp. 37-49.
- 4. P. Nason, S. Dawson and R. K. Ellis, Nucl. Phys. B 303, 607-633 (1988).
- 5. M. L. Mangano, P. Nason and G. Ridolfi, Nucl. Phys. B 373, 295-345 (1992).
- 6. P. Nason, S. Dawson and R. K. Ellis, Nucl. Phys. B 327, 49-92 (1989) [erratum: Nucl. Phys. B 335, 260-260 (1990)]
- 7. G. P. Lepage, J. Comput. Phys. 27, 192 (1978).
- 8. W. Bai and M. H. Reno, J. High Energ. Phys., 2019, 77 (2019).
- 9. W. Bai, M. Diwan, M. V. Garzelli, Y. S. Jeong and M. H. Reno, J. High Energ. Phys. 2020, 032 (2020)
- PROSA collaboration, O. Zenaiev, M. Garzelli, K. Lipka, S. Moch, A. Cooper-Sarkar, F. Olness et al., J. High Energ. Phys. 2020, 118, (2020).